SOLUTION OF EXERCISE # 4.2

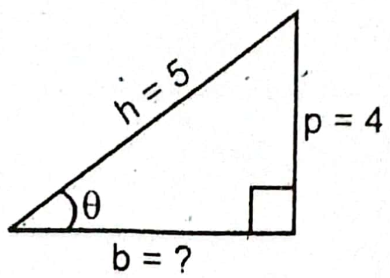
Exercise # 4.2

Q.1: **If $\sin\theta = \frac{4}{5}$ and the terminal ray of θ is in the first quadrant, find the value of :**

(i) $\sin 2\theta$

Sol. As, $\sin\theta = \frac{4}{5}$

By using Pythagoras Theorem



$b^2 + p^2 = h^2$

$b^2 + (4)^2 = (5)^2$

$b^2 = 25 - 16$

$b^2 = 9$

$\sqrt{b^2} = \sqrt{9} \quad \Rightarrow \quad b = 3$

As, 'θ' is in I – Quad, so

$\cos\theta = \frac{b}{h} = \frac{3}{5}$

We know that

$$\sin 2\theta = 2\sin\theta\cos\theta = 2\left(\frac{4}{5}\right)\left(\frac{3}{5}\right) = \boxed{\frac{24}{25}}$$

(ii) $\cos 2\theta$

Sol. $\cos 2\theta = 1 - 2\sin^2\theta = 1 - 2\left(\frac{4}{5}\right)^2$

$$= 1 - 2\left(\frac{16}{25}\right) = 1 - \frac{32}{25} = \frac{25 - 32}{25} = \boxed{-\frac{7}{25}}$$

Q.2: **If $\cos\theta = \frac{4}{5}$ and the terminal ray of θ is in the first quadrant, find the value of:**

(i) $\sin\frac{\theta}{2}$ | **(ii)** $\cos\frac{\theta}{2}$

SOLUTION OF EXERCISE # 4.2

Sol. $\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$

$= \sqrt{\frac{1-\left(\frac{4}{5}\right)}{2}} = \sqrt{\frac{\frac{5-4}{5}}{2}}$

$= \sqrt{\frac{1}{10}} = \boxed{\frac{1}{\sqrt{10}}}$

Sol. $\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$

$= \sqrt{\frac{1+\left(\frac{4}{5}\right)}{2}} = \sqrt{\frac{\frac{5+4}{5}}{2}}$

$= \sqrt{\frac{9}{10}} = \boxed{\frac{3}{\sqrt{10}}}$

(iii) $\tan\frac{\theta}{2}$

Sol. $\tan\frac{\theta}{2} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \frac{\frac{1}{\sqrt{10}}}{\frac{3}{\sqrt{10}}} = \boxed{\frac{1}{3}}$

Q.3: If $\cos\theta = -\frac{5}{13}$ and the terminal side of θ is in the second quadrant, find the value of:

(i) $\sin\frac{\theta}{2}$

Sol. $\sin\frac{\theta}{2} = \sqrt{\frac{1-\cos\theta}{2}}$

$= \sqrt{\frac{1-\left(-\frac{5}{13}\right)}{2}}$

$= \sqrt{\frac{\frac{13+5}{13}}{2}} = \sqrt{\frac{18}{26}}$

$= \sqrt{\frac{9}{13}} = \boxed{\frac{3}{\sqrt{13}}}$

(ii) $\cos\frac{\theta}{2}$

Sol. $\cos\frac{\theta}{2} = \sqrt{\frac{1+\cos\theta}{2}}$

$\cos\left(\frac{\theta}{2}\right) = \sqrt{\frac{1+\left(-\frac{5}{13}\right)}{2}}$

$= \sqrt{\frac{\frac{13-5}{13}}{2}} = \sqrt{\frac{8}{26}}$

$= \sqrt{\frac{4}{13}} = \boxed{\frac{2}{\sqrt{13}}}$

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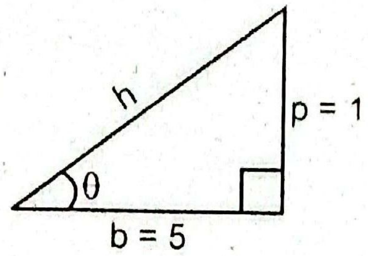
Q.4: **If $\tan\theta = -\frac{1}{5}$, the terminal ray of θ lies in the second quadrant, then find:**

(i) $\sin 2\theta$ **(ii)** $\cos 2\theta$

Sol. As, $\tan\theta = -\frac{1}{5}$

By using Pythagoras Theorem



$b^2 + p^2 = h^2$

$(5)^2 + (1)^2 = h^2$

$25 + 1 = h^2$

$26 = h^2$

$\sqrt{26} = \sqrt{h^2} \Rightarrow h = \sqrt{26}$

As θ lie in II − Quad.

$$\cos\theta = -\frac{b}{h}$$

$$\cos\theta = -\frac{5}{\sqrt{26}}$$

As θ lie in II − Quad.

$$\sin\theta = \frac{p}{h}$$

$$\sin\theta = \frac{1}{\sqrt{26}}$$

As we know

(i) $\sin 2\theta$ **(IA-2022)**

Sol. $\sin 2\theta = 2\sin\theta\cos\theta$

$$\sin 2\theta = 2\left(-\frac{5}{\sqrt{26}}\right)\left(\frac{1}{\sqrt{26}}\right)$$

$$\sin 2\theta = -\frac{10}{26}$$

$$\sin 2\theta = \boxed{-\frac{5}{13}}$$

(ii) $\cos 2\theta$

Sol. $\cos 2\theta = 2\cos^2\theta - 1$

$$= 2\left(\frac{5}{\sqrt{26}}\right)^2 - 1$$

$$= 2\left(\frac{25}{26}\right) - 1$$

$$= \frac{50-26}{26} = \frac{24}{26} = \boxed{\frac{12}{13}}$$

Prove the following identities.

Q.5: $\cos\theta = 2\cos^2\frac{\theta}{2} - 1$

Sol. L.H.S. $= \cos\theta$

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$$= \cos\left(\frac{2\theta}{2}\right) = \cos\left(\frac{\theta+\theta}{2}\right) = \cos\left(\frac{\theta}{2}+\frac{\theta}{2}\right)$$

$$= \cos\frac{\theta}{2}\cos\frac{\theta}{2} - \sin\frac{\theta}{2}\sin\frac{\theta}{2}$$

$$= \cos^2\frac{\theta}{2} - \sin^2\frac{\theta}{2}$$

$$= \cos^2\frac{\theta}{2} - \left(1-\cos^2\frac{\theta}{2}\right)$$

$$= \cos^2\frac{\theta}{2} - 1 + \cos^2\frac{\theta}{2} = 2\cos^2\frac{\theta}{2} - 1 = \text{R.H.S.}$$ **Proved.**

Q.6: $\sin 2\theta = \dfrac{2\tan\theta}{1+\tan^2\theta}$

Sol. L.H.S.= $\sin 2\theta$

$$= 2\sin\theta\cos\theta = \frac{2\sin\theta\cos\theta}{1}$$

$$= \frac{2\sin\theta\cos\theta}{\cos^2\theta+\sin^2\theta} \qquad \because \cos^2\theta+\sin^2\theta = 1$$

Diving numerator and denominator by $\cos^2\theta$

$$= \frac{\dfrac{2\sin\theta\cos\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$$

$$= \frac{2\dfrac{\sin\theta}{\cos\theta}}{\dfrac{\cos^2\theta}{\cos^2\theta}+\dfrac{\sin^2\theta}{\cos^2\theta}} = \frac{2\tan\theta}{1+\tan^2\theta} = \text{R.H.S.}$$ **Proved.**

Q.7: $\cos 2\theta = \dfrac{1-\tan^2\theta}{1+\tan^2\theta}$

Sol. L.H.S.= $\cos 2\theta$

$$= \cos^2\theta - \sin^2\theta = \frac{\cos^2\theta - \sin^2\theta}{1}$$

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$$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta} \qquad \because \cos^2\theta + \sin^2\theta = 1$$

Diving numerator and denominator by $\cos^2\theta$

$$= \frac{\dfrac{\cos^2\theta}{\cos^2\theta} - \dfrac{\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta}{\cos^2\theta} + \dfrac{\sin^2\theta}{\cos^2\theta}} = \frac{1-\tan^2\theta}{1+\tan^2\theta} = \text{R.H.S.} \quad \textbf{Proved.}$$

Q.8: $\tan\theta = \dfrac{\sin 2\theta}{1+\cos 2\theta}$

Sol. $\text{R.H.S.} = \dfrac{\sin 2\theta}{1+\cos 2\theta}$

$$= \frac{2\sin\theta\cos\theta}{1+2\cos^2\theta-1}$$

$$= \frac{2\sin\theta\cos\theta}{2\cos^2\theta}$$

$$= \frac{\sin\theta}{\cos\theta}$$

$$= \tan\theta = \text{L.H.S.}$$

Proved.

Q.9: $\cot\theta = \dfrac{\sin 2\theta}{1-\cos 2\theta}$

Sol. $\text{R.H.S.} = \dfrac{\sin 2\theta}{1-\cos 2\theta}$

$$= \frac{2\sin\theta\cos\theta}{1-(1-2\sin^2\theta)}$$

$$= \frac{2\sin\theta\cos\theta}{1-1+2\sin^2\theta}$$

$$= \frac{2\sin\theta\cos\theta}{2\sin^2\theta}$$

$$= \frac{\cos\theta}{\sin\theta} = \cot\theta = \text{L.H.S.}$$ **Proved.**

Q.10: $\dfrac{1+\sin 2A - \cos 2A}{1+\sin 2A + \cos 2A} = \tan A$

Sol. $\text{L.H.S.} = \dfrac{1+\sin 2A - \cos 2A}{1+\sin 2A + \cos 2A}$

$$= \frac{1+2\sin A\cos A - (1-2\sin^2 A)}{1+2\sin A\cos A + 2\cos^2 A - 1}$$

$$= \frac{1+2\sin A\cos A - 1 + 2\sin^2 A}{1+2\sin A\cos A + 2\cos^2 A - 1}$$

$$= \frac{2\sin A\cos A + 2\sin^2 A}{2\sin A\cos A + 2\cos^2 A}$$

SOLUTION OF EXERCISE # 4.2

$$= \frac{2\sin A(\cos A + \sin A)}{2\cos A(\sin A + \cos A)}$$

$$= \frac{\sin A}{\cos A} = \tan A = \text{R.H.S.}$$ **Proved.**

Q.11: $\sec 2A + \tan 2A = \dfrac{\cos A + \sin A}{\cos A - \sin A}$

Sol. $\text{L.H.S.} = \sec 2A + \tan 2A$

$$= \frac{1}{\cos 2A} + \frac{\sin 2A}{\cos 2A}$$

$$= \frac{1 + \sin 2A}{\cos 2A}$$

$$= \frac{\cos^2 A + \sin^2 A + 2\sin A\cos A}{\cos^2 A - \sin^2 A}$$

$$= \frac{(\cos A + \sin A)^2}{(\cos A - \sin A)(\cos A + \sin A)}$$

$$= \frac{(\cos A + \sin A)}{(\cos A - \sin A)} = \text{R.H.S.}$$ **Proved.**

Q.12: $\operatorname{cosec} 2\theta + \cot 2\theta = \cot\theta$

Sol.

$\text{L.H.S.} = \operatorname{cosec} 2\theta + \cot 2\theta$

$$= \frac{1}{\sin 2\theta} + \frac{\cos 2\theta}{\sin 2\theta}$$

$$= \frac{1 + \cos 2\theta}{\sin 2\theta}$$

$$= \frac{2\cos^2\theta}{2\sin\theta\cos\theta}$$

$$= \frac{\cos\theta}{\sin\theta} = \cot\theta = \text{R.H.S.}$$

Proved.

Q.13: $\dfrac{1 - \cos 2A}{1 + \cos 2A} = \tan^2 A$

Sol. $\text{L.H.S.} = \dfrac{1 - \cos 2A}{1 + \cos 2A}$

$$= \frac{1 - (1 - 2\sin^2 A)}{1 + (2\cos^2 A - 1)}$$

$$= \frac{1 - 1 + 2\sin^2 A}{1 + 2\cos^2 - 1}$$

$$= \frac{2\sin^2 A}{2\cos^2 A} = \tan^2 A = \text{R.H.S.}$$

Proved.

SOLUTION OF EXERCISE # 4.2

Q.14: $\dfrac{\sin 3\theta}{\cos\theta}+\dfrac{\cos 3\theta}{\sin\theta}=2\cot 2\theta$

Sol. $\text{L.H.S.}=\dfrac{\sin 3\theta}{\cos\theta}+\dfrac{\cos 3\theta}{\sin\theta}$

$=\dfrac{\sin 3\theta\sin\theta+\cos 3\theta\cos\theta}{\cos\theta\sin\theta}$

$=\dfrac{\cos 3\theta\cos\theta+\sin 3\theta\sin\theta}{\sin\theta\cos\theta}$

$=\dfrac{2\cos(3\theta-\theta)}{2\sin\theta\cos\theta}$

$=\dfrac{2\cos 2\theta}{\sin 2\theta}=2\cot 2\theta=\text{R.H.S.}$

Proved.

Q.15: $\dfrac{\cot^2\theta-1}{\operatorname{cosec}^2\theta}=\cos 2\theta$

Sol. $\text{L.H.S.}=\dfrac{\cot^2\theta-1}{\operatorname{cosec}^2\theta}$

$=\left(\dfrac{\cos^2\theta}{\sin^2\theta}-1\right)\sin^2\theta$

$=\left(\dfrac{\cos^2\theta-\sin^2\theta}{\sin^2\theta}\right)\sin^2\theta$

$=\cos^2\theta-\sin^2\theta$

$=\cos 2\theta=\text{R.H.S.}$

Proved.

Q.16: $\cos^4\theta-\sin^4\theta=\dfrac{1}{\sec 2\theta}$ **(IIA-2017)**

Sol. L.H.S.

$=\cos^4\theta-\sin^4\theta$

$=(\cos^2\theta)^2-(\sin^2\theta)^2$

$=(\cos^2\theta-\sin^2\theta)(\cos^2\theta+\sin^2\theta)$

$=(\cos 2\theta)(1)=\dfrac{1}{\sec 2\theta}=\text{R.H.S.}$

Proved.

Q.17: $\left(\sin\dfrac{\theta}{2}+\cos\dfrac{\theta}{2}\right)^2=1+\sin\theta$

Sol. L.H.S.=

$\left(\sin\dfrac{\theta}{2}+\cos\dfrac{\theta}{2}\right)^2$

$=\sin^2\dfrac{\theta}{2}+\cos^2\dfrac{\theta}{2}+2\sin\dfrac{\theta}{2}\cos\dfrac{\theta}{2}$

$=1+\sin\theta=\text{R.H.S.}$

Proved.

Q.18: $(\sin\theta-\cos\theta)^2=1-\sin 2\theta$ **(IIA-2019)**

Sol. $\text{L.H.S.}=(\sin\theta-\cos\theta)^2$

$=\sin^2\theta+\cos^2\theta-2\sin\theta\cos\theta$

$=1-\sin 2\theta=\text{R.H.S.}$ **Proved.**

Q.19: $\sin^4\theta=\dfrac{3}{8}-\dfrac{1}{2}\cos 2\theta+\dfrac{1}{8}\cos 4\theta$ **(IIA-2018)**

Sol. $\text{L.H.S.}=\sin^4\theta=(\sin^2\theta)^2$

SOLUTION OF EXERCISE # 4.2

$$=\left(\frac{1-\cos 2\theta}{2}\right)^2 \qquad \because \sin^2\theta=\frac{1-\cos 2\theta}{2}$$

$$=\frac{(1)^2-2(1)\cos 2\theta+(\cos 2\theta)^2}{4}$$

$$=\frac{1}{4}[1-2\cos 2\theta+\cos^2 2\theta]$$

$$=\frac{1}{4}\left[1-2\cos 2\theta+\frac{1+\cos 4\theta}{2}\right] \qquad \because \cos 2\theta=\frac{1+\cos 4\theta}{2}$$

$$=\frac{1}{4}\left[\frac{2-4\cos 2\theta+1+\cos 4\theta}{2}\right]$$

$$=\frac{1}{8}[3-4\cos 2\theta+\cos 4\theta]$$

$$=\frac{3}{8}-\frac{4\cos 2\theta}{8}+\frac{\cos 4\theta}{8}$$

$$=\frac{3}{8}-\frac{\cos 2\theta}{2}+\frac{\cos 4\theta}{8}=\text{R.H.S.}$$ **Proved.**

Q.20: Compute the value of $\sin\frac{\pi}{12}$ from the function of $\frac{\pi}{6}$.

Sol. $$\sin^2\frac{\pi}{12}=\frac{1-\cos\frac{\pi}{6}}{2}=\frac{1-\frac{\sqrt{3}}{2}}{2}=\frac{\frac{2-\sqrt{3}}{2}}{2}=\frac{2-\sqrt{3}}{4}$$

taking squaring root on both sides

$$\sqrt{\sin^2\frac{\pi}{2}}=\sqrt{\frac{2-\sqrt{3}}{4}}$$

$$\sin\frac{\pi}{12}=\boxed{\frac{\sqrt{2-\sqrt{3}}}{2}}$$